**Section 3.4.1 – Odds ratios**

Odds ratios are easily formed because the proportional odds regression model equates log-odds to the linear predictor. The main difference now is the odds involve cumulative probabilities.

Consider the model again of



The odds ratio is



where  denotes the odds of observing category j or smaller for Y. The formal interpretation of the odds ratio is

The odds of Y ≤ j vs. Y > j are exp(cβ1) times as large for a c-unit increase in x.

We can also use

The odds of Y ≤ j vs. Y > j change by exp(cβ1) times for every c-unit increase in x.

**Interestingly, this odds ratio stays the same no matter what response category is used for j.** This is again due the absence of a j subscript on β1 in the model. We can then modify the interpretations to be

The odds of Y being below a particular level are exp(cβ1) times as large for a c-unit increase in x.

or

The odds of Y being below a particular level change by exp(cβ1) times for every c-unit increase in x.

Notes:

* When there is more than one explanatory variable, we will need to include a statement like “holding the other variables in the model constant”.
* Similar to what we saw in Chapter 2, adjustments need to be made for an odds ratio calculation and interpretation when interactions or transformations are present in the model.
* Inference methods for odds ratios are performed in the same ways as for likelihood procedures discussed in earlier chapters.

Example: Wheat kernels (Wheat.R, Wheat.csv)

The estimated odds ratios for each explanatory variable are calculated as  for r = 1, …, 6. As done for the multinomial regression model, c is set equal to one standard deviation for each explanatory variable, except for class which uses c = 1. Below are the calculations (remember that  is ):

> sd.wheat <- apply(X = wheat[,-c(1,7,8)], MARGIN = 2, FUN = sd)

> c.value <-c(1, sd.wheat)

> round(c.value, 2) # class = 1 is first value

density hardness size weight moisture

1.00 0.13 27.36 0.49 7.92 2.03

> round(exp(c.value \* (-mod.fit.ord$coefficients)), 2)

density hardness size weight moisture

0.84 0.17 0.75 1.15 0.37 1.08

> round(1/exp(c.value \* (-mod.fit.ord$coefficients)), 2)

density hardness size weight moisture

1.19 5.89 1.33 0.87 2.74 0.92

Example interpretations include:

* The estimated odds of a scab (Y ≤ 1) vs. sprout or healthy (Y > 1) response are 0.84 times as large for soft rather than for hard red winter wheat, holding the other variables constant. Note that the corresponding 95% confidence interval for the class contains 1 as we will see shortly.
* The estimated odds of a scab vs. sprout or healthy response are 5.89 times as large for a 0.13 decrease in the density, holding the other variables constant.
* The estimated odds of a scab vs. sprout or healthy response are 2.74 times as large for a 7.92 decrease in the weight, holding the other variables constant.

Because of the proportional odds, each of the previous interpretations can start with “The estimated odds of a scab or sprout vs. healthy response are ... ,” and the same estimated odds ratios would be used in the interpretation. Also, one could put the interpretation in the following form:

The estimated odds of kernel quality being below a particular level are \_\_\_ times as large for a \_\_\_ increase in \_\_\_, holding the other variables constant.

due to the proportional odds.

Overall, we see that the larger the density and weight, the more likely a kernel is higher quality. We can again relate these results back to parallel coordinates plot to see why these interpretations make sense.

Below are the calculations for the profile likelihood ratio confidence intervals:

> conf.beta <- confint(object = mod.fit.ord, level = 0.95)

Waiting for profiling to be done...

Re-fitting to get Hessian

> ci <- exp(c.value\*(-conf.beta))

> round(data.frame(low = ci[,2], up = ci[,1]), 2)

low up

classsrw 0.39 1.81

density 0.11 0.26

hardness 0.55 1.03

size 0.77 1.72

weight 0.23 0.58

moisture 0.76 1.54

> round(data.frame(low = 1/ci[,1], up = 1/ci[,2]), 2)

low up

classsrw 0.55 2.57

density 3.87 9.36

hardness 0.97 1.83

size 0.58 1.29

weight 1.73 4.40

moisture 0.65 1.31

Note that the c.value\*(-conf.beta) segment of code multiplies each row of conf.beta by its corresponding element in c.value.

The density odds ratio can be interpreted as:

With 95% confidence, the odds of a scab instead of a sprout or healthy response are between 3.87 to 9.36 times as large when the density is decreased by 0.13, holding the other variables constant.

Similar modifications to the above interpretation can be made as before due to the proportional odds assumption. For example,

With 95% confidence, the estimated odds of kernel quality being below a particular level are between 3.87 and 9.36 times as large when the density is decreased by 0.13, holding the other variables constant.

Wald intervals cannot be calculated using confint.default(). Instead, here are the calculations for density using emmeans:

> calc.est <- emmeans(object = mod.fit.ord, specs = ~ density, at = list(density = c(1.25 + 0.13, 1.25)), mode = "linear.predictor")

Re-fitting to get Hessian

> confint(object = contrast(object = calc.est, method = "revpairwise", type = "response"), level = 0.95)

contrast odds.ratio SE df asymp.LCL

density1.25 / density1.38 5.79 1.29 Inf 3.74

asymp.UCL

8.95

Results are averaged over the levels of: class, cut

Confidence level used: 0.95

Intervals are back-transformed from the log odds ratio scale

Comments:

* Odds ratios can be calculated for only one explanatory variable at a time.
* Differences from using emmeans with multinomial regression:
  + The specs argument contains ONLY the explanatory variable of interest.
  + The mode value is linear.predictor.
  + The method argument replaces the interaction argument of contrast().