Project #2

STAT 878

Spring 2024

Complete the problems below. Within each part, include your R program output with code inside of it and any additional information needed to explain your answer. Your R code and output should be formatted in the exact same manner as in the course notes.

1. (19 total points) This problem continues working with the earthquakes data from Project #1.
   1. (3 points) Plot the estimated ACF and estimated PACF. Use lags of 1 to 20 and set y-axis limits of -1 to 1. Put these plots in 1 row and 2 columns within the same plotting window.
   2. (3 points) What is ? Perform a hypothesis test for H0: ϕ22 = 0 vs. Ha: ϕ22 ≠ 0. Make sure to fully state all numerical values needed for the test and a conclusion.
   3. (3 points) Based on the plots in a), suggest two possible models for the data. Justify your suggestions using the “cuts off to 0” and “tails off to 0” phrasing given in the course notes.
   4. Use an ARIMA(1,0,1) model for this part. Note that p = q = 1 may NOT be a correct answer for part c)!!!
      1. (5 points) Estimate and state an ARIMA(1,0,1) model.
      2. (5 points) Perform the hypothesis tests of H0: ϕ1 = 0 vs. Ha: ϕ1 ≠ 0 and H0: θ1 = 0 vs. Ha: θ1 ≠ 0 using confidence intervals.
2. (23 total points) The purpose of this problem is to see the effect that the sample size has on determining an appropriate ARMA(p,q) model for data. Suppose ϕ1 = 0.5789, θ1 = -0.1598, and  = 1. When ACF and PACF plots are constructed, use lags of 1 to 20 and set y-axis limits of -1 to 1. Put these plots in 1 row and 2 columns within the same plotting window.
   1. (3 points) Is the model causal and/or invertible? Explain.
   2. (3 points) Plot the true ACF and PACF. Comment on how well the plots follow the patterns expected for an ARMA(1,1) model as described in the course notes.
   3. Simulate one time series with n = 25 using arima.sim(). Set a seed of 7831 right before simulating the data.
      1. (3 points) Print the data. Credit for this problem is also being given for the data simulation.
      2. (3 points) Plot the estimated ACF and estimated PACF for the data.
      3. (3 points) Comment on how well the plots follow the patterns expected for an ARMA(1,1) model as described in the course notes.
   4. (3 points) Construct one function that will repeat the data simulation and plotting process in part c). Arguments for this function should be values for ϕ1, θ1, n, , and the seed number. Run the function using n = 25 to verify that the function is correct.
   5. (5 points) Run the function in d) using n = 100, 1000, and 10000 and the same seed as before. Construct estimated ACF and PACF plots for each sample size. Compare these plots to those in b).
3. (11 total points) For an ARMA(2,1) with μ = 0, complete the following.
   1. (5 points) Develop an expression that writes the model as an infinite order moving average with coefficients denoted by ψk. Once you find the pattern for ψk, you may express ψk as a function of ψk-1, ψk-2, … , rather than expressing it in terms of the autoregressive and moving average parameters alone.
   2. (3 points) Suppose ϕ1 = 0.6, ϕ2 = 0.2, and θ1 = -0.4. Using the results from a), construct your own R function that finds ψk for k = 1, …, 20. Run the function and list out these values.
   3. (3 points) Use the ARMAtoMA() function to perform the same calculations as in b).